MAHARAJA INSTITUTE OF TECHNOLOGY MYSORE
(An Autonomous Institution)
Belawadi, Srirangapatna Taluk, Mandya-571477
DEPARTMENT OF MATHEMATICS

Module-1

1. Find the angle of intersection between the curves $r^2 sin2\theta = 4$ and $r^2 = 16sin2\theta.$
2. With usual notation prove that tan$\emptyset = r\frac{d\theta}{dr}$
3. With usual notation prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$
4. Find the angle between the curves $r = alog\theta$, $r = \frac{\theta}{log\theta}$
5. Show that the radius of curvature at any point of the cycloid $x = a(\theta + sin\theta), = a(1 - cos\,\theta)$ is 4acos($\frac{\theta}{2}$).
6. Show that the curves $r = a\,(1 + \sin\theta)$, $r = b(1 - sin\theta)$ cut each other orthogonally.
7. Find the pedal equation of the curve $\frac{2a}{r} = (1 + cos\theta)$.
8. Using modern mathematical tool write a program/code to plot thecurve $r = 2|cos2\theta|$.
9. Find the angle between the curves,$r = \frac{a}{1+cos\theta}$,$r = \frac{b}{1-cos\theta}$.
10. Find the radius of curvature of the curve $y = x^3(x - a)$ at the point(a,0).
11. Show that the curves $r = a\,(1 + cos\,\theta\,)$, $r = b(1 - cos\theta)$ cut each other orthogonally.
12. Find the pedal equation of the curve $r(1 - cos\theta) = 2a$.
13. Using modern mathematical tool write a program/code to plot thesine and cosine curve.
14. Find the radius of curvature for the curve $y^2 = \frac{4a^2(2a-x)}{x}$ where the curve meets the x-axis.
15. Find the angle of intersection between the curves $r^2 sin2\theta = 4$ and $r^2 = 16sin2\theta.$
16. For the curve $r = a(1 - cos\theta)$, $\frac{p^2}{r}$ is a constant.
17. Find the angle between radius vector and tangent for the curve$r = a(1 + cos\theta)$ and also find the slope of the tangent at $\theta = \frac{\pi}{3}$.
18. Find the angle of intersection between the curves $r = \frac{a\theta}{1} + \theta$ and $r = \frac{\theta}{(1+\theta^2)}$.
19. Find the pedal equation of the curve $r^n = a(1 + cosn\theta)$.
20. Find the pedal equation of the curve $r^m = a^m \cos(m\theta)$.
21. Find the pedal equation $r^m = a^m\, sinm\theta + b^m cosm\theta$
22. Show that the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $(\frac{3a}{2}, \frac{3a}{2})$.
23. Show that the evaluate of the parabola $y^2 = 4ax\ is\ 27ay^2 = 4(x - 2a)^3$.
24. Find the radius of curvature of the point (3a,3a) on the curve x3+y3=3axy.
25. Show that the radius of curvature.
26. For the catenary $y = \cosh\left(\frac{x}{c}\right)$ at any point (x, y) varies as square ofthe ordinate at that point.

MAHARAJA INSTITUTE OF TECHNOLOGY MYSORE

(An Autonomous Institution)

Belawadi, Srirangapatna Taluk, Mandya-571477

DEPARTMENT OF MATHEMATICS

MODULE-02: DIFFERENTIAL CALCULUS-2

Problems on Maclaurin's series expansion:

1. Find e^x upto the term containing x^4.
2. Find $\sin x$ and $\cos x$ upto the term containing x^4
3. Expand $\log(1+x)$ upto the term containing x^4
4. Find the first four non-zero terms in the expansion $y(x) = \frac{x}{e^{x-1}}$ using Maclaurin's series
5. Expand $\sqrt{1+\sin 2x}$ $upto$ x^4
6. Expand $\log(\sec x)$ in ascending powers of x upto the first three non-vanishing terms.
7. Expand $\log(\cos x)$ $upto\ the\ term\ containing\ x^6$.
8. Expand $\log(1+\cos x)$ $upto\ the\ term\ containing\ x^4$.
9. Expand $e^{\sin x}$ $upto\ the\ term\ containing\ x^6$.
10. Expand $e^{x\sin x}$ $upto\ the\ term\ containing\ x^4$.
11. Expand $e^{\cos x}$ $upto\ the\ term\ containing\ x^4$.
12. Expand $e^{x\cos x}$ $upto\ the\ term\ containing\ x^4$.

Problems on Indeterminate form of $1^\infty, 0^0, \infty^0, 0^\infty$

1. $\underset{x \to 0}{Lt} \left[\frac{a^x+b^x}{2}\right]^{1/x}$
2. $\underset{x \to 0}{Lt} \left[\frac{2^x+3^x}{2}\right]^{1/x}$
3. $\underset{x \to 0}{Lt} \left[\frac{2^x+3^x+4^x}{3}\right]^{1/x}$
4. $\underset{x \to 1}{Lt} \left(x^{\frac{1}{1-x}}\right)$
5. $\underset{x \to 0}{Lt} \left(\frac{\tan x}{x}\right)^{1/x}$
6. $\underset{x \to 0}{Lt} (\cos x)^{1/x^2}$
7. $\underset{x \to 0}{Lt} \left(\frac{\sin x}{x}\right)^{1/x}$
8. $\underset{x \to 0}{Lt} \left(\frac{\sin x}{x}\right)^{1/x^2}$
9. $\underset{x \to 0}{Lt} \left(\frac{\tan x}{x}\right)^{1/x^2}$
10. $\underset{x \to 0}{Lt} \left(\frac{ax+1}{ax-1}\right)^{x}$
11. $\underset{x \to 0}{Lt} \tan x^{\tan x}$
12. $\underset{x \to \infty}{Lt} \left(1+\frac{1}{x}\right)^{x}$

MAHARAJA INSTITUTE OF TECHNOLOGY MYSORE
(An Autonomous Institution)
Belawadi, Srirangapatna Taluk, Mandya-571477
DEPARTMENT OF MATHEMATICS

Problems on composite functions:

1. $u = x^2y + xy^2$ *where* $x = at, y = 2at$ then find $\frac{du}{dt}$
2. If $u = sin\left(\frac{x}{y}\right)$ *where* $x = e^t$ & $y = t^2$
3. If $u = e^x\, sin(yz)$ *where* $x = t^2, y = t - 1, z = \frac{1}{t}$ *at* $t = 1$
4. If $u = x^2 + y^2 + z^2$ *where* $x = e^t, y = e^t \cos t$ & $z = e^t \sin t$
5. If $z = f(x, y)$ where $x = e^u + e^{-v}$ & $y = e^{-u} - e^v$
6. If Z=f(x,y) where $x = e^u \cos v$ & $y = e^u \sin v$ then P.T
$$\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = e^{2u}\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right]$$
7. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ *then P.T* $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
8. **If** $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then P.T $\frac{1}{2}u_x + \frac{1}{3}u_y + \frac{1}{4}u_z = 0$
9. If $u = f\left(xz, \frac{y}{z}\right)$ then S.T $xu_x - yu_y - zu_z = 0$
10. If $u = f(x^2 + 2yz, y^2 + 2zx)$ then P.T $(y^2 - zx)u_x + (x^2 - yz)u_y + (z^2 - xy)u_z = 0$
11. Using modern mathematical tool write a program/code to S.T. $u_{xx} + u_{yy} + u_{zz}$=0 given u=$e^x(xcosy - ysiny)$
12. If u= f (ax-by, by-cz, cz-ax) P.T. $\frac{1}{a}\frac{\partial u}{\partial x} + \frac{1}{b}\frac{\partial u}{\partial y} + \frac{1}{c}\frac{\partial u}{\partial z}$ =0
13. If u= f(2x-3y, 3y-4z, 4z-2x) S.T. 6 $\frac{\partial u}{\partial x}$ + 4 $\frac{\partial u}{\partial y}$ + 3$\frac{\partial u}{\partial z}$=0
14. If z=$\frac{x^2+y^2}{x+y}$ S.T. $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$
15. If u=$e^{(ax+by)}$f(ax-by) P.T. b $\frac{\partial u}{\partial x}$ +a $\frac{\partial u}{\partial y}$= 2abu by using composite functions.
16. If u= $tan^{-1}\left(\frac{y}{x}\right)$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

Problems on Jacobians:

1. If $u = x + y + z,\ v = y + z, w = z$, then find its Jacobian.
2. If $x = r \sin\theta \cos\phi, y = r \sin\theta \sin\phi, z = r \cos\theta$, then find J.
3. If $x = u^2 - v^2, y = v^2 - w^2, z = w^2 - u^2$, then find J
4. If $u = xy^2, v = yz^2$ & $\omega = zx^2$,then find its Jacobian.
5. If x+y+z=u, y+z=uv, z=uvw. Find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

Problems on Maxima and Minima:

1. Find the extreme value for the following functions $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$
2. Divide the number 24 into 3 parts such that product may be maximum.
3. A rectangular box is open at the top is to have volume 108 cubic meters. Find its dimension so that total surface area is minimum.

4. S.T f(x,y)=1 + $sin(x^2 + y^2)$ is minimum at(0,0)
5. Find the minimum value of $x^2 + y^2 + z^2$ when x+y+z=3a
6. Find the minimum value of $u = x^2 + y^2 + z^2$ when $xyz = a^3$
7. Find the stationary value of the function xy+yz+zx subject to the condition x+y+z=1
8. Examine the function f (x , y)=2(x^2 - y^2)- x^4 + y^4 for the extreme values.
9. Find the extreme values of f (x , y) =x^3 +3x^2+4xy+y^2
10. S.T the function f (x, y) =x^3 + y^3-3xy+1 is minimum at a point (1,1)

MODULE-3

Ordinary Differential Equations (ODEs) of first Order

1) Solve $\left(y^2e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0.$
2) Solve $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0.$
3) Solve $(x^2 + y^3 + 6x)dx + y^2xdy = 0.$
4) Solve $x\frac{dy}{dx} + y = x^3y^6$
5) Solve $\frac{dy}{dx} = xy^3 - xy.$
6) Solve $\frac{dy}{dx} - ytanx = y^2secx.$
7) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0.$
8) Solve $xy(1 + xy^2)\frac{dy}{dx} = 1.$
9) Solve $(x^2 - 4xy - 2y^2)dx$ +$(y^2 - 4xy - 2x^2)dy = 0.$
10) Show that the orthogonal trajectories of a family of circles pasing through the origin having centres on x-axis is a family of circles passing through the origin having their centres on y-axis.
11) Find the orthogonal trajectories of a family of curves $r = a(1 + cos\theta)$.
12) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda}$=1 Where λ is a parameter.
13) Find the orthogonal trajectories of a family of curves $r^n = a^n \cos n\theta$.
14) Find the orthogonal trajectories of a family of curves $r^n = a^n \sin n\theta$.
15) Show that the orthogonal trajectories of a family of $r = acos^2\frac{\theta}{2}$ is another family of $r = bsin^2\frac{\theta}{2}$.
16) A body is heated to 110^0C and placed in air at 10^0C. After one hour its temperature become 60^0C. How much additional time is required for it to cool to 30^0C.
17) Suppose that an object is heated to 300^0 F and allowed to cool in a room whose air temperature is 80^0F. After 10 minutes the temperature of the object is 250^0 F. What will be its temperature after 20 minute
18) If a substance cools from 370k to 330K in 10 minutes , when the temperature of the surrounding air is 290k.Find the temperature of the substance after 40minutes
19) If the temperature of the air is 30^0c and the substance cools from 100^0c to 70^0c in 15minutes , find when the temperature reaches at 40^0c.

20) If the temperature of the air is $30^0\,C$ and a metal ball cools from $100^0\,C$ to $70^0\,C$ in 15minutes,
find how long will it take for the metal ball to reach a temperature of $40^0\,C$.
21) Solve $y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$
22) Solve the equation $(px-y)(py+x) = 2p$ by reducing into clairaut's form , Taking the substitution X= x^2 Y=y^2.
23) Find the general & singular solution of the equation $xp^2 - yp + a = 0$.
24) Solve $y = 2px + p^2y$.
25) Solve $p(p+y) = x(x+y)$.
26) Solve $p^2 + 2pycotx = y^2$
27) Find the orthogonal trajectories of a family of curves $r^n cosn\theta = a^n$,where a is a parameter.
28) Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$.
29) A copper ball originally ay 80^0C cools down to 60^0C in 20 minutes, if the temperature of the air being 40^0C. What will be the temperature of the ball after 40mnutes from the original.
30) If the temperatue of the air is 30^0C and a metal ball cools from 100^0C to 70^0C in 15 minutes, Find how long will it take for the metal ball to reach at temperatue of 40^0C.
31) Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

32) Solve$[rsin\theta - r^2]d\theta - [cos\theta]dr = 0$.
33) Solve $dy + [xsin2y - x^3cos^2y]dx = 0$
34) Solve $p^4 - [x+2y+1]p^3 + [x+2y+2xy]p^2 - 2xyp = 0$
35) Find the general and singular solution of $[px-y][x-py] = 2p$ by using substitution $x^2 = u\ ,y^2 = v$
36) The damped LCR circuit is governed by the equation $L\,\frac{d^2q}{dt^2} + R\,\frac{dq}{dt} + \frac{q}{c} = 0\ where, L, C, R$ are positive constants. Find the conditions under which the circuit is over damped, under damped and critically damped. Find also the critical resistance.
37) The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L\,\frac{d^2q}{dt^2} + \frac{i}{C} = 0$. Find the current i as a function of t given that I is the maximum current, and $i = 0$ when $t = 0$.
38) An alternating $E.M.F.E\ \sin pt$ is applied to a circuit at $t = 0$. Given the equation for the current i as $L\,\frac{d^2i}{dt^2} + R\,\frac{di}{dt} + \frac{i}{c} = pE\ \cos pt$, find the current i when i) $CR^2 > 4L$, ii) $CR^2 < 4L$.

MAHARAJA INSTITUTE OF TECHNOLOGY MYSORE
(An Autonomous Institution)
Belawadi, Srirangapatna Taluk, Mandya-571477
DEPARTMENT OF MATHEMATICS

Module-4: Modular Arithmetic(CSE stream)

Introduction of modular arithmetic and its applications in Computer Science and Engineering.

1) Find the reminder when 2^{23} is divided by 47.
2) Find the reminder when 2^{50} is divided by 7.
3) Find the reminder when 2^{1000} is divided by 13.
4) Find the reminder when 14! is divided by 17.
5) Find the reminder when 15! is divided by 17
6) Show that 4(29)!+5! is divisible by 31.
7) Find the reminder when 2(26)! is divisible by 29.
8) Find the last digit in 7^{18}.
9) Solve $3^{202} mod\ 13$ by Euler's theorem.
10) Solve $4^{99} mod\ 35$ by Euler's theorem.
11) Use Euler's theorem to find the unit digit in 3^{100}.
12) Find the solutions of the linear congruence $11x \equiv 4(mod 25)$.
13) Encrypt the message **STOP** using RSA with key (2537, 13) using the prime numbers 43 and 59.
14) Using Fermat's Little Theorem, show that $8^{30} - 1$ is divided by 31.
15) Find the reminder when 72^{1001} is divided by 31.
16) Show that $2^{340} \equiv 1\ mod\ 31$ by Fermat's Little Theorem.
17) Find $7^{121}\ mod\ 13$.
18) Find the reminder when 41^{75} is divided by 3.
19) Find the reminder when 5^{11} is divided by 7.
20) Show that $5^{38} \equiv 4 mod(11)$.
21) Solve the system of linear congruence $x \equiv 3(mod 5),\ y \equiv 2(mod 6), z \equiv 4(mod 7)$ using Remainder Theorem.
22) Solve the following system of equations using Chinese Remainder Theorem $x \equiv 5(mod 3),\ x \equiv 2(mod 5), x \equiv 1(mod 11)$.
23) Solve the following system of equations using Chinese Remainder Theorem $x \equiv 2(mod 3),\ x \equiv 3(mod 5), x \equiv 2(mod 7)$.
24) Find the remainder when 175*113*53 is divided by 11.
25) Solve $x^3 + 5x + 1 \equiv 0(mod)27$.
26) Find the remainder when 349*74*35 is divided by 3.
27) Find the remainder when 135*74*48 is divided by 7.
28) Find the last digit in 7^{2013}
29) Find the last unit digit in 7^{289}
30) Find the last unit digit in 7^{126}
31) Find the last digit in 13^{27}

32) Find the least positive values of x such that i) $71 \equiv x(mod 8)$ ii) $78 + x \equiv 3(mod 5)$ iii) $89 \equiv (x + 3)(mod 4)$.
33) Solve $2x + 6y \equiv 1(mod 7)$, $4x + 3y \equiv 2(mod 7)$.
34) Solve $5x + 6y \equiv 10(mod 13)$, $6x - 7y \equiv 2(mod 13)$
35) Find the GCD of 32 & 54 and express it in the form $32x + 54y$.
36) Find the least positive values of x such that $i)$ $5x \equiv 4(mod 6)$ $ii)$ $7x \equiv 9(mod 15)$
37) Determine all the solutions in the positive integers of the linear Diophantine equation 54x+21y=906.
38) Find the general solution of the equation$70x + 112y = 168$.
39) Find the general solution of the equation $39x - 56y = 11$.
40) Using RSA algorithm find public key and private key w.r.to $p = 3$, $q = 11$ & $M = 31$.
41) In RSA algorithm if $p = 7, q = 11$ & $e = 13$ then what will be the value of d?
42) If $p = 3, Q = 11$ and private key $d = 7$ find the public key using RSA algorithm and hence encrypt the number 19.

Module-4

Ordinary Differential Equations of Higher Order(CV & ME Stream)

Homogeneous Differential Equation :

1. Solve: $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$, given $x(0) = 0,\ \frac{dx}{dt}(0) = 15$
2. Solve: $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$
3. Solve: $(D^2 - 2D + 4)^2 y = 0$
4. Solve: $(D^2 + 1)^3 y = 0$
5. Solve: $(D^3 + D^2 + 4D + 4) = 0$
6. Solve: $\frac{d^4x}{dt^4} + 4x = 0$

Solve:

7. $\frac{d^2x}{dt^2} + 3a\frac{dx}{dt} - 4a^2x = 0$
8. $y'' - 2y' + 10y = 0,\ y(0) = 4,\ y'(0) = 1$
9. $4y''' + 4y'' + y' = 0$
10. $\frac{d^3y}{dx^3} + y = 0$

11. $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$
12. $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$
13. $(D^2+1)^2(D-1)y = 0$
14. If $\frac{d^4x}{dt^4} = m^4x$, show that $x = c_1\cos mt + c_2\sin mt + c_3\cosh mt + c_4\sinh mt$
15. Solve: $\frac{d^4y}{dx^4} + a^4y = 0$

Homogeneous Differential Equation :

1. Solve: $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$
2. Solve: $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$
3. Solve: $(D^3 - 3D + 2)y = 0$
4. Solve: $4y''' + 4y'' + y' = 0$
5. Solve: $\frac{d^3y}{dx^3} + y = 0$
6. Solve: $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$
7. Solve: $(D^4 - 5D^2 + 4)y = 0$
8. Solve: $\frac{d^4y}{dt^4} + 8\frac{d^2y}{dt^2} + 16y = 0$
9. Solve: $(D^4 + 64)y = 0$
10. Solve: $\frac{d^4x}{dt^4} - 2\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} = 0$
11. Solve: $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$
12. Solve: $(D^5 - D^4 - D + 1)y = 0$
13. Solve: $y'' + 4y' + 4y = 0$ given that $y = 0$, $y' = -1$ at $x = 1$
14. Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ given that $y = 2$, and $\frac{dy}{dx} = \frac{d^2y}{dx^2}$ when $x = 0$
15. Solve: $\left\{D^2(D^2+2D)^2(D^2+2D+2)^3\right\}y = 0$

16. Solve: $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$
17. Solve: $\frac{d^3y}{dx^3} - 8y = 0$

18. Solve: $(D^3 - 3D^2 + 4)x = 0$, where $D = \frac{d}{dt}$
19. Solve: $16y''' - 8y'' + y' = 0$
20. Solve: $2x'''(t) + 5x''(t) - 12x'(t) = 0$
21. Solve: $(D^4 - 2D^3 + 2D^2 - 2D + 1)y = 0$
22. Solve: $y'' - 4y' + 5y = 0$ subject to the conditions $y'(0) = 2, \quad y(0) = 1$

Non-Homogeneous Differential equation

Type – 1:

1. Find the P.I of $(D^2 + 5D + 6)y = e^x$

Type – 2:

1. Find the P.I of $(D^3 + 1)y = \cos(2x - 1)$
2. Find the P.I of $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$

Type – 3:

1. Find the P.I of $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$

Solve:

1. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$
2. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$. Also find y when $y = 0, \ \frac{dy}{dx} = 1 \ at \ x = 0$
3. $\frac{d^2x}{dt^2} + n^2 x = k\cos(nt + \alpha)$
4. $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = \sin t$
5. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$
6. $(D^2 - 4D + 3)y = \sin 3x \cos 2x$
7. $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$
8. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$
9. $\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 3^x$
10. $\frac{d^2y}{dx^2} + 4y = x^2 + \cos 2x$
11. $(D^3 - D)y = 2x + 1 + 4\cos x + 2e^x$

12. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = e^{2x} + \sin x + x$
13. $(D^2+1)^2 y = x^4 + 2\sin x \cos 3x$
14. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sin 2x$
15. $(D^4 + D^2 + 1)y = e^{-x/2}\cos\frac{\sqrt{3}}{2}x$
16. $(D^4 - 1)y = e^x \cos x$
17. $(D^2 + 4D + 3)y = e^{-x}\sin x + xe^{3x}$
18. $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x\cos 2x$
19. $\frac{d^4y}{dx^4} - y = \cos x \cosh x$
20. $(D^3 + 2D^2 + D)y = x^2e^{2x} + \sin^2 x$
21. $\frac{d^2y}{dx^2} + 4y = x\sin x$
22. $(D^2 + 2D + 1)y = x\cos x$
23. $(D^2 - 1)y = x\sin x + (1+x^2)e^x$
24. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$
25. $(D^2 + a^2)y = \tan ax$

Non-Homogeneous Differential equation

Type – 1:

1. Solve: $6\frac{d^2y}{dx^2} + 17\frac{dy}{dx} + 12y = e^{-x}$
2. Solve: $y'' + 2y' + y = \cosh\left(x/2\right)$
3. Solve: $(D^3 - D^2 + 4D - 4)y = \sinh(2x+3)$
4. Solve: $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = 5e^{-2t}$
5. Solve: $\frac{d^4x}{dt^4} + 4x = \cosh t$
6. Solve: $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$
7. Solve: $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 13y = e^{3t}\cosh 2t + 2^t$

8. Solve: $(D^4-18D^2+81)y=36e^{3x}$

9. Solve: $\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+3\frac{dy}{dx}+y=5e^{2x}+6e^{-x}+7$

10. Solve: $y''-6y'+13y=e^{2x}+2^x$

11. Solve: $4x''(t)-x(t)=e^{t/2}+12\cosh t$

12. Solve: $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=6e^{3x}+7e^{-2x}-\log 2$

13. Solve: $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=-2\cosh x$. Also find y when $y=0,\ \frac{dy}{dx}=1\ \ at\ \ x=0.$

Type – 2:

1. Solve: $y''-4y'+13y=\cos 2x$
2. Solve: $y''+9y=\cos 2x.\cos x$
3. Solve: $(D^3-1)y=3\cos 2x$
4. Solve: $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=4\cos^2 x$
5. Solve: $(D^2+4)y=\sin^2 x$
6. Solve: $D^2(D^2+4)(D^2+9)y=2\sin\left(x/2\right)\cos\left(x/2\right)$
7. Solve: $(D^4+8D^2+16)y=2\cos^2 x$
8. Solve: $\frac{d^3y}{dx^3}+y=65\cos(2x+1)$
9. Solve: $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y+37\sin 3x=0$, find the value of y when $x=\pi/2$ if it is given that $y=3$ and $\frac{dy}{dx}=0$ when $x=0.$

10. Solve: $y''-3y'+2y=2\sin x\cos x$
11. Solve: $y'''-3y''+9y'-27y=\cos 3x$
12. Solve: $x''(t)+8x'(t)+25x(t)=16(3\cos t-\sin t)$
13. Solve: $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+3x=\sin t$
14. Solve: $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=4\cos^2 x$
15. Solve: $(D^2-4D+3)y=\sin 3x\cos 2x$

Type – 3

1. Solve: $y''+2y'+y=2x+x^2$

2. Solve: $(D^3+8)y = x^4+2x+1$

3. Solve: $\frac{d^3x}{dt^3}+3\frac{d^2x}{dt^2}=1+t$

4. Solve: $(D^2+3D+2)y=1+3x+x^2$

5. Solve: $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=x^2$

6. Solve: $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^3$

7. Solve: $y''+y'+y=x^2+x+1$

8. Solve: $\frac{d^3y}{dx^3}-8y=x(x^2+1)$

9. Solve: $x'''(t)-x''(t)-6x'(t)=1+t^2$

Method of Variation of Parameters

1. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+4y=\tan 2x$
2. Solve, by the method of variation of parameters, $\frac{d^2y}{dx^2}-y=\frac{2}{(1+e^x)}$
3. Solve by the method of variation of parameters $y''-6y'+9y=e^{3x}/x^2$
4. Solve, by the method of variation of parameters, $y''-2y'+y=e^x\log x$

Solve by the method of variation of parameters:

5. $\frac{d^2y}{dx^2}+y=\operatorname{cosec} x$
6. $\frac{d^2y}{dx^2}+a^2y=\sec ax$
7. $\frac{d^2y}{dx^2}+y=\tan x$
8. $\frac{d^2y}{dx^2}+y=x\sin x$
9. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x/x$
10. $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}$
11. $y''-2y'+2y=e^x\tan x$

12. Solve by the method of variation of parameters $y'' + a^2 y = \sec ax$.
13. Solve: $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameters.
14. Solve $\frac{d^2y}{dx^2} + y = \sec x \tan x$ by the method of variation of parameters.
15. Solve $(D^2+1)y = \operatorname{cosec} x \cot x$ by the method of variation of parameters.
16. Solve by the method of variation of parameters $y'' + 4y = 4\sec^2 2x$.
17 By the method of variation of parameters solve : $y'' - 2y' + y = e^x \log x$.
18. Using the method of variation of parameters solve : $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$.
19. Solve by the method of variation of parameters $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$.
20. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.
21. Solve by the method of variation of parameters $y'' + 2y' + 2y = e^{-x}\sec^3 x$.
22. Solve $(D^2 - 3D + 2)y = \cos(e^{-x})$ by the method of variation of parameters.
23. Solve $(D^2 + 3D + 2)y = e^{e^x}$ by the method of variation of parameters.

<u>Cauchy's and Legendre's LDE</u>

1. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$
2. Solve $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$
3. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$
4. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + y = \log x \frac{\sin(\log x)+1}{x}$
5. Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$
6. Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$
7. Solve $(2x-1)^2\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} - 2y = 8x^2 - 2x + 3$

Solve:

8. $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x^2$

9. $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^4$

10. $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=(1+x)^2$

11. $x\frac{d^2y}{dx^2}-\frac{2y}{x}=x+\frac{1}{x^2}$

12. Solve $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$

13. Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\log x$

14. Solve $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x)$

15. Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$

16. Solve $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$

17. Solve $x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+4y=x\log x$

18. Solve $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-12y=x^3\log x$

19. Solve $(2x+3)^2\frac{d^2y}{dx^2}-(2x+3)\frac{dy}{dx}-12y=6x$

20. Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log(1+x)$

21. Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\sin[2\log(1+x)]$

22. Solve $(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1$

23. Solve: $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=(1+x)^2$

24. Solve: $x^2y''+xy'+9y=3x^2+\sin(3\log x)$

25. Solve: $x\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=\frac{1}{x}$

26. Solve: $x\frac{d^2y}{dx^2}-\frac{2y}{x}=x+\frac{1}{x^2}$

27. Solve the Legendre's form of linear equation
$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\sin 2[\log(1+x)]$$

28. Solve: $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin[\log(\overline{1+x})]$

29. Solve: $(2x+1)^2y''-6(2x+1)y'+16y=8(2x+1)^2$

30. Solve: $x^2y''-xy'+2y=x\sin(\log x)$

31. Solve: $x^4\frac{d^3y}{dx^3}+2x^3\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=\sin(\log x)$

32. Solve: $x^2y''-xy'+y=x^2\log x$

33. Solve: $x^2\frac{d^2y}{dx^2}-(2m-1)x\frac{dy}{dx}+(m^2+n^2)y=n^2x^m\log x$

34. Solve the Cauchy's linear equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\sin(\log x)$

35. Solve: $x^2D^2y-3xDy+5y=x^2\sin(\log x)$

36. Solve: $(2x+1)^2y''-2(2x+1)y'-12y=6x+5$

37. Solve: $(3x+2)^2y''+3(3x+2)y'-36y=8x^2+4x+1$

Solve the following equations

38. $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$

39. $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$

40. $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=\sin(\log x)$

41. $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-12y=x^2\log x$

42. $2xy''+3y'-\frac{y}{x}=5-\frac{\sin(\log x)}{x^2}$

43. $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log(1+x)$

44. $(2x+1)^2y''-2(2x+1)y'-12y=x\log(2x+1)$

45. $(3x-2)^2y''-3(3x-2)y'=9(3x-2)\sin\log(3x-2)$

46. $(x+2)^2y''-(x+2)y'+y=3x+4$

MODULE-4

Double and Triple Integrals(ECE Stream)

1. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$
2. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$
3. Evaluate $\int_0^1 \int_x^{\sqrt{1-y^2}} x^3 y dx dy$
4. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2) dz dy dx$
5. Evaluate $\int_{-1}^{1} \int_{-0}^{z} \int_{x-z}^{x+z} (x + y + z) dy dx\, dz$
6. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz dy dx$
7. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{\sqrt{1-x^2-y^2-z^2}}$
8. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz dy dx$
9. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{\frac{(a^2-r^2)}{a}} r\, dr\, d\theta dz$
10. Evaluate $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dy dx dz$
11. Evaluate $\int_0^{log2} \int_0^x \int_0^{x+logy} e^{x+y+z}\, dz dy dx$
12. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz dy dx}{(1+x+y+z)^3}$
13. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ by changing the order of integration
14. Change the order of the integration and hence evaluate $\int_0^1 \int_{\sqrt{y}}^1 dx\, dy$
15. Evaluate by changing the order of integration $\int_0^1 \int_x^1 \frac{x\, dy dx}{\sqrt{x^2+y^2}} dy\, dx$
16. Change the order of the integration and hence evaluate $\int_0^{4a} \int_{y=\frac{x^2}{4a}}^{y=2\sqrt{ax}} xy\, dy dx$
17. Evaluate by changing the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$
18. Evaluate by changing the order of integration $\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} (2 - x) dy\, dx$
19. Evaluate by changing the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dx\, dy$
20. Change the order of the integration and hence evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\, dy dx$
21. Change the order of the integration and hence evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\, dx\, dy$
22. Change the order of the integration and hence evaluate $\int_1^2 \int_1^{x^2} (x^2 + y^2)\, dy dx$

Evaluation by Changing into Polars

23. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$ by changing to polar coordinates
24. Change the integral $\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dydx$ into polar and hence evaluate the same
25. Evaluate $\int_0^a \int_x^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\, dx\, dy$ by changing into polar
26. Evaluate $\int_0^a \int_x^{2\sqrt{ax}} x^2\, dx\, dy$ by changing into polar

Applications of Double and Triple integrals

27. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration
28. Find by double integration the area enclosed by the curve $r=a(1+cos\theta)$ between $\theta=0$ to $\theta=\pi$
29. Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0, \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
30. Find the volume generated by the revolution of the $r=a(1+cos\theta)$ about the initial line
31. Using the double integration find the area between the parabolas $y^2=4ax$ and $x^2=4ay$.

Beta and Gamma Functions

32. Prove that relation between beta and gamma function $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
33. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$
34. By definition of gamma function prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$
35. Show that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{Sin\theta}} \times \int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\ d\theta = \pi$
36. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{cot\theta}\ d\theta$ by expressing in terms of gamma functions
37. Evaluate $\int_0^{\frac{\pi}{2}} sin^6\theta\ d\theta$
38. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta}\ d\theta$ by expressing in terms of gamma functions
39. Evaluate $\int_0^{\frac{\pi}{2}} cos^7\theta\ d\theta$
40. Evaluate $\int_0^{\frac{\pi}{2}} sin^4\theta\ cos^3\theta d\theta$
41. Evaluate $\int_0^\infty \frac{x}{1+x^6}\ dx$
42. Evaluate $\int_0^\infty \frac{dx}{1+x^4}\ dx$
43. Evaluate $\int_0^2 (4-x^2)^{3/2}\ dx$
44. Evaluate $\int_0^1 x^{3/2}(1-x)^{1/2}\ dx$

MODULE-5

Linear Algebra

1. Find the Rank of the following matrices by applying elementary row transformations

$$\begin{bmatrix}1 & 2 & 3 & 2\\ 2 & 3 & 5 & 1\\ 1 & 3 & 4 & 5\end{bmatrix}, \begin{bmatrix}-2 & -1 & -3 & -1\\ 1 & 2 & 3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}, \begin{bmatrix}1 & 2 & 3 & 0\\ 2 & 4 & 3 & 2\\ 3 & 2 & 1 & 3\\ 6 & 8 & 7 & 5\end{bmatrix}$$

2. Find the rank of the matrix $\begin{bmatrix}4 & 0 & 2 & 1\\ 2 & 1 & 3 & 4\\ 2 & 3 & 4 & 7\\ 2 & 1 & 3 & 4\end{bmatrix}$ using elementary row operation by reducing it to echelon form.

3. Reduce the matrix $\begin{bmatrix}1 & 2 & 1 & 3 & 4\\ 2 & 1 & 3 & 2 & 1\\ 0 & 2 & 1 & 1 & 3\\ 3 & 1 & 3 & 4 & 2\end{bmatrix}$ to the echelon form and find its rank.

4. Find the values of k such that the following matrix A may have rank equal to (i)3 (ii)2

$$A = \begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 2 & 4 & k\\ 1 & 4 & 10 & k^2\end{bmatrix}$$

5. Reduce the following matrix to the Echelon form and find its rank -

$$A = \begin{bmatrix}0 & 1 & 2 & -2\\ 4 & 0 & 2 & 6\\ 2 & 1 & 3 & 1\end{bmatrix}$$

$$A = \begin{bmatrix}8 & 1 & 3 & 6\\ 0 & 3 & 2 & 2\\ -8 & -1 & -3 & 4\end{bmatrix}$$

$$A = \begin{bmatrix}1 & 0 & 2 & 1\\ 0 & 1 & -2 & 1\\ 1 & -1 & 4 & 0\\ -2 & 2 & 6 & 0\end{bmatrix}$$

$$A = \begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{bmatrix}$$

$$A = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 12 & 13 & 14 & 15 \\ 13 & 14 & 15 & 16 \\ 14 & 15 & 16 & 17 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

6. Test for consistency and solve:

$$\begin{aligned} x + y + z &= 6 \\ x - y + 2z &= 5 \\ 3x + y + z &= 8 \end{aligned}$$

7. Show that the system of equations

$$\begin{aligned} x + y + z &= 4 \\ 2x + y - z &= 1 \\ x - y + 2z &= 2 \end{aligned}$$

Is consistent and hence find the solution.

8. Test for consistency and solve:

$$\begin{aligned} x + 2y + 3z &= 14 \\ 4x + 5y + 7z &= 35 \\ 3x + 3y + 4z &= 21 \end{aligned}$$

9. Test for consistency and solve:

$$\begin{aligned} 5x + 3y + 7z &= 4 \\ 3x + 26y + 2z &= 9 \\ 7x + 2y + 10z &= 5 \end{aligned}$$

10. Show that the following system of equations does not possess any solution

$$\begin{aligned} 5x + 3y + 7z &= 5 \\ 3x + 26y + 2z &= 9 \\ 7x + 2y + 10z &= 5 \end{aligned}$$

11. Investigate the values of λ and μ such that the system of equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$$

may have

(i) Unique solution (ii) Infinite solution (iii) No solution

12. Find for what values of k the system of equations

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 4z &= k \\ x + 4y + 10z &= k^2 \end{aligned}$$

possesses a solution.

Solve completely in each case.

13. Test for consistency and solve:

$$x+y+z=-3$$
$$3x+y-2z=-2$$
$$2x+4y+7z=7$$

14. Test for consistency and solve:

$$x+y+z=9$$
$$2x+5y+7z=52$$
$$2x+y-z=0$$

15. Investigate the values of λ and μ such that the system of equations

$$2x+3y+5z=9$$
$$7x+3y-2z=8$$
$$2x+3y+\lambda z=\mu \quad \text{may have}$$

(i) Unique solution (ii) Infinite solution (iii) No solution

16. Solve by Gauss elimination method

$$x+y+z=4$$
$$2x+y-z=1$$
$$x-y+2z=2$$

17. Solve by Gauss elimination method

$$2x_1+x_2+4x_3=12$$
$$4x_1+11x_2-x_3=33$$
$$8x_1-3x_2+2x_3=20$$

18. Solve by Gauss elimination method

$$x_1+x_2+x_3+4x_4=-6$$
$$x_1+7x_2+x_3+x_4=12$$
$$x_1+x_2+6x_3+x_4=-5$$
$$5x_1+x_2+x_3+x_4=4$$

19. Solve by Gauss elimination method

$$2x_1-x_2+3x_3=1$$
$$-3x_1+4x_2-5x_3=0$$
$$x_1+3x_2-6x_3=0$$

20. Solve by Gauss elimination method

$$4x_1+x_2+x_3=4$$
$$x_1+4x_2-2x_3=4$$
$$3x_1+2x_2-4x_3=6$$

21. Solve by Gauss Jordon method

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

22. Solve by Gauss Jordon method

$$x + y + z = 9$$
$$2x + y - z = 0$$
$$2x + 5y + 7z = 52$$

23. Solve by Gauss Jordon method

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

24. Solve by Gauss Jordon method

$$x_1 + x_2 + x_3 + x_4 = 2$$
$$2x_1 - x_2 + 2x_3 - x_4 = -5$$
$$3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$$
$$x_1 - 2x_2 - 3x_3 + 2x_4 = 5$$

25. Solve by Gauss elimination method

$$x_1 - 2x_2 + 3x_3 = 2$$
$$3x_1 - x_2 + 4x_3 = 4$$
$$2x_1 + x_2 - 2x_3 = 5$$

26. Solve the system of linear equations by Gauss Seidel method

$$27x + 6y - z = 85,\ 6x + 15y + 2z = 72,\ x + y + 54z = 110$$

27. Solve the following system of the equation by Gauss Seidel iterative method

$$10x - 2y - z - w = 3,\ -2x + 10y - z - w = 15,\ -x - y + 10z - 2w = 27,\ -x - y - 2z + 10w = -9$$

28. Solve by Guass seidel method

1. $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 0$

2. $28x + 4y - z = 32$, $2x + 17y + 4z = 35$, $x + 3y + 10z = 24$

3. $x + 2y + z = 3,\ 2x + 3y + 3z = 10,\ 3x - y + 2z = 13$

4. $2x + 3y - z = 5,\ 4x + 4y - 3z = 3,\ 2x - 3y + 2z = 2$

5. $3x + 4y + 5z = 18,\ 2x - y + 8z = 13,\ 5x - 2y + 7z = 20$

29. Determine the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ using the power method.

30. Determine the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using the power method.

31. Determine the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ using the power method.

32. Determine the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ using the power method.

33. Determine the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ using the power method with initial approximation $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

34. Determine the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ using the power method.

35. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form and hence find A^4

36. Diagonalize the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$

37. Diagonalize the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

38. Diagonalize the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ and hence find A^5